

北一女中 105 學年度《數戰數決》有獎徵答活動

班別： 一 年 良 班 座號： 17 號 姓名： 黃品淳

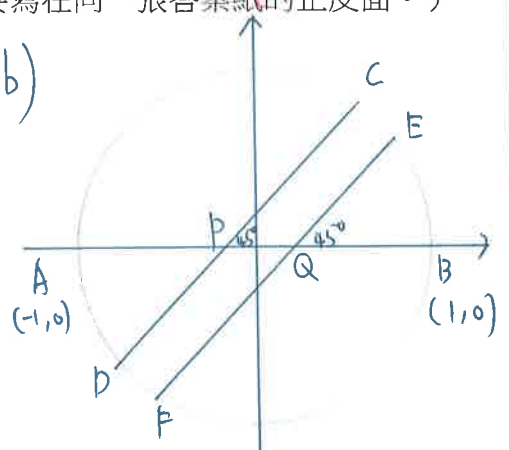
題號： 5 頁碼/總頁數： (如果只有一頁，可不填)

(請不要將兩題的解答寫在同一張答案紙，一題的解答也不要寫在同一張答案紙的正反面。)

$$\text{令 } P = (a, 0), Q = (b, 0) \quad (-1 < a, b < 1 \wedge a \neq b)$$

$$\vec{CD} : y = x - a$$

$$\vec{EF} : y = x - b$$



$$C, D : \begin{cases} y = x - a \\ x^2 + y^2 = 1 \end{cases}$$

$$\Rightarrow x^2 + (x-a)^2 = 1 \\ 2x^2 - 2ax + a^2 - 1 = 0$$

$$\Rightarrow x = \frac{a \pm \sqrt{2-a^2}}{2}, y = \sqrt{1-x^2}$$

$$C \left(\frac{a + \sqrt{2-a^2}}{2}, \sqrt{\frac{1-a\sqrt{2-a^2}}{2}} \right) \quad D \left(\frac{a - \sqrt{2-a^2}}{2}, -\sqrt{\frac{1+a\sqrt{2-a^2}}{2}} \right)$$

$$E, F : \begin{cases} y = x - b \\ x^2 + y^2 = 1 \end{cases}$$

$$\Rightarrow E \left(\frac{b + \sqrt{2-b^2}}{2}, \sqrt{\frac{1-b\sqrt{2-b^2}}{2}} \right) \quad F \left(\frac{b - \sqrt{2-b^2}}{2}, -\sqrt{\frac{1+b\sqrt{2-b^2}}{2}} \right)$$

$$F \left(\frac{b - \sqrt{2-b^2}}{2}, -\sqrt{\frac{1+b\sqrt{2-b^2}}{2}} \right)$$

$$\overline{CP}^2 = \left(\frac{\sqrt{2-a^2} - a}{2} \right)^2 + \frac{1-a\sqrt{2-a^2}}{2} = \frac{2-2a\sqrt{2-a^2}+2-2a\sqrt{2-a^2}}{4} = 1-a\sqrt{2-a^2}$$

$$\overline{QE}^2 = \left(\frac{\sqrt{2-b^2} - b}{2} \right)^2 + \frac{1-b\sqrt{2-b^2}}{2} = 1-b\sqrt{2-b^2}$$

$$\overline{CP} \cdot \overline{QE} = \sqrt{(1-a\sqrt{2-a^2})(1-b\sqrt{2-b^2})} < \frac{2-a\sqrt{2-a^2}-b\sqrt{2-b^2}}{2} \quad (\because a \neq b)$$

$$\overline{PD}^2 = \left(\frac{-a - \sqrt{2-a^2}}{2} \right)^2 + \frac{1+a\sqrt{2-a^2}}{2} = \frac{2a\sqrt{2-a^2}+2+2+2a\sqrt{2-a^2}}{4} = 1+a\sqrt{2-a^2}$$

$$\overline{QF}^2 = 1+b\sqrt{2-b^2}$$

$$\overline{PC} \cdot \overline{QE} + \overline{PD} \cdot \overline{QF} < \frac{2-a\sqrt{2-a^2}-b\sqrt{2-b^2}}{2} + \frac{2+a\sqrt{2-a^2}+b\sqrt{2-b^2}}{2}$$

$$\overline{PD} \cdot \overline{QF} = \sqrt{(1+a\sqrt{2-a^2})(1+b\sqrt{2-b^2})} < \frac{2+a\sqrt{2-a^2}+b\sqrt{2-b^2}}{2}$$

$$= 2$$

110 110 110