

北一女中 105 學年度《數戰數決》有獎徵答活動

班別：一年良班 座號：17 號 姓名：黃品淳

題號：6 頁碼/總頁數： (如果只有一頁，可不填)

(請不要將兩題的解答寫在同一張答案紙，一題的解答也不要寫在同一張答案紙的正反面。)

令 w, x, y, z 為一方程式之四根

$$w + x + y + z = 10$$

$$\frac{(w+x+y+z)^2 - (w^2+x^2+y^2+z^2)}{2} = \frac{10^2 - 30}{2} = 35 = wx + wy + wz + xy + xz + yz$$

$$\begin{aligned} (w+x+y+z)^3 &= (w+x+y+z) \left[w^2+x^2+y^2+z^2 + 2(wx+wy+wz+xy+xz+yz) \right] \\ &= w^3+x^3+y^3+z^3 + 3(w^2x+w^2y+w^2z+x^2w+x^2y+x^2z+y^2w+y^2x+y^2z+z^2w+z^2x+z^2y) + 6(wxy+wyz+wxz+xyw) \end{aligned}$$

$$10^3 = 100 + 3(w^2+x^2+y^2+z^2)(w+x+y+z) - 3(w^3+x^3+y^3+z^3) + 6(wxy+wyz+wxz+xyw)$$

$$1000 = 100 + 3 \times 30 \times 10 - 3 \times 100 + 6 \times (\text{三三乘積和})$$

$$wxy + wxz + wyz + xyw = \frac{300}{6} = 50$$

$$\begin{aligned} (t-w)(t-x)(t-y)(t-z) &= t^4 - 10t^3 + 35t^2 - 50t + 24 \\ &= (t-1)(t-2)(t-3)(t-4) \end{aligned}$$

$$\therefore w=1, x=2, y=3, z=4$$

$$A: (w, x, y, z) = (1, 2, 3, 4)$$