

北一女中 105 學年度《數戰數決》有獎徵答活動

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題號：1 頁碼/總頁數：1 (如果只有一頁，可不填)

(請不要將兩題的解答寫在同一張答案紙，一題的解答也不要寫在同一張答案紙的正反面。)

$$\text{Cpf)} \sqrt{2\sqrt{3\cdots\sqrt{2016\sqrt{2017}}}} < 3$$

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$$\text{令 } X = 2^{\frac{1}{2}} \times 3^{\frac{1}{2^2}} \times 4^{\frac{1}{2^3}} \cdots \times 2016^{\frac{1}{2^{2015}}} \times 2017^{\frac{1}{2^{2016}}}$$

$$\text{取 } \log \downarrow \log X = \frac{1}{2} \log 2 + \frac{1}{2^2} \log 3 + \cdots + \frac{1}{2^{2015}} \log 2016 + \frac{1}{2^{2016}} \log 2017$$

$$\text{key} \uparrow 2 \log X = \log 2 + \frac{1}{2} \log 3 + \frac{1}{2^2} \log 4 + \cdots + \frac{1}{2^{2015}} \log 2017 \rightarrow \text{互減}$$

$$\Rightarrow \log X = \log 2 + \frac{1}{2} (\log 3 - \log 2) + \frac{1}{2^2} (\log 4 - \log 3) + \cdots + \frac{1}{2^{2015}} (\log 2017 - \log 2016)$$

$$\Rightarrow \log X < \log 2 + \frac{1}{2} (\log 3 - \log 2) + \frac{1}{2^2} (\log 4 - \log 3) + \cdots + \frac{1}{2^{2015}} (\log 2017 - \log 2016) + \frac{1}{2^{2016}} \log 2017$$

$$\leq \log 2 + \frac{1}{2} (\log 3 - \log 2) + \frac{1}{2^2} (\log 3 - \log 2) + \cdots + \frac{1}{2^{2015}} (\log 3 - \log 2)$$

$$= \log 2 + [(\log 3 - \log 2) (\frac{1}{2} + \frac{1}{2^2} + \cdots + \frac{1}{2^{2015}})]$$

$$= \log 2 + [(\log 3 - \log 2) (\frac{\frac{1}{2} [1 - (\frac{1}{2})^{2015}]}{1 - \frac{1}{2}})] \quad \text{等比公式}$$

$$= \log 2 + [(\log 3 - \log 2) (1 - (\frac{1}{2})^{2015})] < 1$$

$$< \log 2 + (\log 3 - \log 2) = \log 3$$

$$\text{i.e. } \log X < \log 3 \Rightarrow \text{原式 } X < 3 \quad \#$$

~~(log 3 - log 2) > (log 4 - log 3) > ...~~

原證明之