

北一女中 103 學年度《數戰數決》有獎徵答活動

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題號： 3-6 頁碼/總頁數： 6/6 (如果只有一頁，可不填)

設 $\overline{AB} = c$, $\overline{AC} = b$, $\overline{BC} = a$

已知 $\angle PAB = \angle BCA$, $\angle CAQ = \angle ABC$

則 $\triangle ABP \sim \triangle CAQ \sim \triangle CBA$ (AA 相似)

$\Rightarrow \overline{AP} = b \times \frac{c}{a} = c \times \frac{b}{a} = \overline{AQ}$, $\overline{BP} = c \times \frac{c}{a}$, $\overline{CQ} = b \times \frac{b}{a}$

又 $\frac{\overline{AQ}}{\overline{AN}} = \frac{\overline{AP}}{\overline{AM}} = \frac{\overline{CP}}{\overline{NM}} = \frac{1}{2} \Rightarrow \overline{AP} \parallel \overline{BC} \parallel \overline{NM}$

$\overline{NM} = (\overline{BP} + \overline{CQ} - \overline{BC}) \times 2 = 2 \left(\frac{b^2 + c^2 - a^2}{a} \right)$

$\therefore \angle ACB = \angle BAM$

$\frac{b^2 + a^2 - c^2}{2ac} = \frac{c^2 + \left(\frac{bc}{a}\right)^2 - \overline{BM}^2}{2 \times c \times \frac{bc}{a}} \Rightarrow \overline{BM}^2 = \frac{c^2(b^2 + 2c^2 - a^2)}{a^2} \Rightarrow \overline{BM} = \frac{c\sqrt{b^2 + 2c^2 - a^2}}{a}$

$\therefore \triangle BCS \sim \triangle MNS$

$\overline{BS} = \overline{BM} \times \frac{\overline{BC}}{\overline{BC} + \overline{NM}} = \frac{c\sqrt{b^2 + 2c^2 - a^2}}{a} \times \frac{a}{\left(\frac{b^2 + c^2 - a^2}{a}\right) + a} = \frac{ac}{\sqrt{b^2 + 2c^2 - a^2}}$

同理 $\overline{CS} = \frac{ab}{\sqrt{b^2 + 2c^2 - a^2}}$

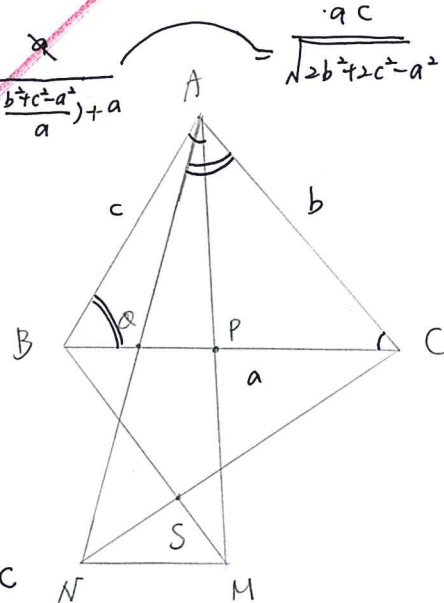
$\cos \angle BSC = \frac{a^2 c^2}{2b^2 + 2c^2 - a^2} + \frac{a^2 b^2}{2b^2 + 2c^2 - a^2} - a^2$
 $= \frac{2 \times \frac{ac}{\sqrt{b^2 + 2c^2 - a^2}} \times \frac{ab}{\sqrt{b^2 + 2c^2 - a^2}}}{2bc} = \frac{a^2 - b^2 - c^2}{2bc}$
 $= - \left(\frac{b^2 + c^2 - a^2}{2bc} \right) = -\cos \angle BAC$

$\therefore 0^\circ \leq \angle BSC, \angle BAC \leq 180^\circ$

$\therefore \angle BSC + \angle BAC = 180^\circ$

$\angle MBA + \angle SCA = 180^\circ$

$\Rightarrow ABCS$ 四共圓 \neq



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