北一女中 102 學年度《數戰數決》有獎徵答活動

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題號：1-5　　頁碼／總頁數：＿＿＿＿（如果只有一頁，可不填）

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$$\begin{cases} x^3-5xy^2=21 \quad \text{—①} \\ y^3-5x^2y=28 \quad \text{—②} \end{cases}$$

①×4－②×3得 $4x^3-20xy^2-3y^3+15x^2y=0$

$$\Rightarrow x^3+\frac{15}{4}x^2y-5xy^2-\frac{3}{4}y^3=0$$

$$\Rightarrow \left(\frac{x}{y}\right)^3+\frac{15}{4}\left(\frac{x}{y}\right)^2-5\left(\frac{x}{y}\right)-\frac{3}{4}=0$$

上式的三根為 $\frac{\alpha_1}{\beta_1}$，$\frac{\alpha_2}{\beta_2}$，$\frac{\alpha_3}{\beta_3}$

$$\Rightarrow \left(\frac{x}{y}\right)^3+\frac{15}{4}\left(\frac{x}{y}\right)^2-5\left(\frac{x}{y}\right)-\frac{3}{4}=\left(\frac{x}{y}-\frac{\alpha_1}{\beta_1}\right)\left(\frac{x}{y}-\frac{\alpha_2}{\beta_2}\right)\left(\frac{x}{y}-\frac{\alpha_3}{\beta_3}\right)$$

令 $\frac{x}{y}=-4$ 代入得

$$\frac{61}{4}=\left(-4-\frac{\alpha_1}{\beta_1}\right)\left(-4-\frac{\alpha_2}{\beta_2}\right)\left(-4-\frac{\alpha_3}{\beta_3}\right)$$

$$\Rightarrow \left(4+\frac{\alpha_1}{\beta_1}\right)\left(4+\frac{\alpha_2}{\beta_2}\right)\left(4+\frac{\alpha_3}{\beta_3}\right)=-\frac{61}{4}$$

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