

北一女中 102 學年度《數戰數決》有獎徵答活動

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題號：1-5 頁碼/總頁數：_____ (如果只有一頁，可不填)

$$\begin{cases} x^3 - 5xy^2 = 21 & \text{--- ①} \\ y^3 - 5x^2y = 28 & \text{--- ②} \end{cases}$$

①×4 - ②×3 得 $4x^3 - 20xy^2 - 3y^3 + 15x^2y = 0$

$$\Rightarrow x^3 + \frac{15}{4}x^2y - 5xy^2 - \frac{3}{4}y^3 = 0$$

$$\Rightarrow \left(\frac{x}{y}\right)^3 + \frac{15}{4}\left(\frac{x}{y}\right)^2 - 5\left(\frac{x}{y}\right) - \frac{3}{4} = 0$$

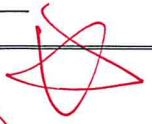
上式的三根為 $\frac{\alpha_1}{\beta_1}, \frac{\alpha_2}{\beta_2}, \frac{\alpha_3}{\beta_3}$

$$\Rightarrow \left(\frac{x}{y}\right)^3 + \frac{15}{4}\left(\frac{x}{y}\right)^2 - 5\left(\frac{x}{y}\right) - \frac{3}{4} = \left(\frac{x}{y} - \frac{\alpha_1}{\beta_1}\right)\left(\frac{x}{y} - \frac{\alpha_2}{\beta_2}\right)\left(\frac{x}{y} - \frac{\alpha_3}{\beta_3}\right)$$

令 $\frac{x}{y} = -4$ 代入得

$$\frac{61}{4} = \left(-4 - \frac{\alpha_1}{\beta_1}\right)\left(-4 - \frac{\alpha_2}{\beta_2}\right)\left(-4 - \frac{\alpha_3}{\beta_3}\right)$$

$$\Rightarrow \left(4 + \frac{\alpha_1}{\beta_1}\right)\left(4 + \frac{\alpha_2}{\beta_2}\right)\left(4 + \frac{\alpha_3}{\beta_3}\right) = \frac{61}{4}$$



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