

北一女中 102 學年度《數戰數決》有獎徵答活動

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題號：1-4 頁碼/總頁數：_____ (如果只有一頁，可不填)

$$a_{n+1}^2 - a_n^2 = \frac{1}{(n+2)^2} - \frac{1}{n^2}$$

$$a_2^2 - a_1^2 = \frac{1}{3^2} - \frac{1}{1^2}$$

$$a_3^2 - a_2^2 = \frac{1}{4^2} - \frac{1}{2^2}$$

$$a_4^2 - a_3^2 = \frac{1}{5^2} - \frac{1}{3^2}$$

⋮

$$+) a_n^2 - a_{n-1}^2 = \frac{1}{(n+1)^2} - \frac{1}{(n-1)^2}$$

$$a_n^2 - a_1^2 = \frac{1}{(n+1)^2} + \frac{1}{n^2} - \frac{1}{1^2} - \frac{1}{2^2}$$

$$a_n^2 = \frac{1}{n^2} + \frac{1}{(n+1)^2} - \frac{5}{4} + \frac{9}{4} = \frac{n^2 + (n+1)^2 + n^2(n+1)^2}{n^2(n+1)^2}$$

$$= \frac{n^4 + 2n^3 + 3n^2 + 2n + 1}{n^2(n+1)^2} = \frac{(n^2 + n + 1)^2}{n^2(n+1)^2}$$

$$\Rightarrow a_n = \frac{n^2 + n + 1}{n(n+1)} = 1 + \frac{1}{n(n+1)} = 1 + \frac{1}{n} - \frac{1}{n+1}$$

$$\sum_{k=1}^{2014} a_k = (1 + \frac{1}{1} - \frac{1}{2}) + (1 + \frac{1}{2} - \frac{1}{3}) + (1 + \frac{1}{3} - \frac{1}{4}) + \dots + (1 + \frac{1}{2014} - \frac{1}{2015})$$

$$= 2014 + 1 - \frac{1}{2015} = 2014 \frac{2014}{2015}$$

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