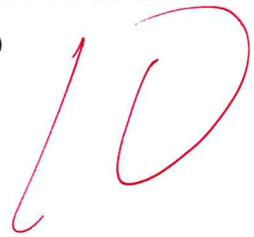


北一女中 102 學年度《數戰數決》有獎徵答活動

班別： 一年 良 班 座號： 13 號 姓名： 林采韻



題號： 1-4 頁碼/總頁數： 4/6 (如果只有一頁，可不填)



$$a_{n+1}^2 = a_n^2 - \frac{1}{n^2} + \frac{1}{(n+1)^2} + \frac{1}{(n+1)^2} - \frac{1}{(n+1)^2}$$

$$a_{n+1}^2 - \frac{1}{(n+1)^2} - \frac{1}{(n+1)^2} = a_n^2 - \frac{1}{n^2} - \frac{1}{(n+1)^2}$$

$$\text{令 } b_n = a_n^2 - \frac{1}{n^2} - \frac{1}{(n+1)^2}$$

$$b_{n+1} = b_n$$

$$b_1 = \frac{2^2}{2} - \frac{1}{1} - \frac{1}{4} = 1$$

$$1 = a_n^2 - \frac{1}{n^2} - \frac{1}{(n+1)^2}$$

$$a_n^2 = 1 + \frac{1}{n^2} + \frac{1}{(n+1)^2}$$

$$a_n = \sqrt{\frac{n^2(n+1)^2 + (n+1)^2 + n^2}{n^2(n+1)^2}} = \sqrt{\frac{n^4 + 2n^3 + n^2 + n^2 + 2n + 1 + n^2}{n^2(n+1)^2}}$$

$$= \frac{n^2 + n + 1}{n(n+1)} = \frac{n}{n+1} + \frac{1}{n}$$

$$\sum_{k=1}^{2014} a_k = \frac{1}{1} + \frac{1}{2} + \frac{1}{2} + \frac{2}{3} + \frac{1}{3} + \frac{3}{4} + \frac{1}{4} + \dots + \frac{1}{2014} + \frac{2014}{2015}$$

$$= 1 + 1 + 1 + 1 + \dots + 1 + \frac{2014}{2015}$$

$$= \frac{2014 \cdot 2014}{2015}$$

#